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LARGE DEFLECTIONS OF RECTANGULAR HOFF SANDWICH PLATES

ZHEN-QIANG CHENG, XIU-XI WANG and MAO-GUANG HUANG
Department of Modern Mechanics, University of Science and Technology of China, Hefei, Anhui 230026, P.R. China

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Abstract—The nonlinear theory of Hoff type sandwich plates underlying geometrically nonlinear dynamic response is derived from Hamilton's principle. It is shown how the fundamental equations and boundary conditions of dimensionless form can be simplified to the Reissner–Mindlin type theory of moderately thick plates, the Reissner's theory of sandwich plates, and the Kirchhoff theory of thin plates. Nonlinear bending of rectangular sandwich plates is investigated under lateral pressure with some symmetric boundary conditions. Exact solutions of series form are obtained by developing a new technique of mixed Fourier series in nonlinear analysis. The nonlinear partial differential equations are reduced to an infinite set of simultaneous nonlinear algebraic equations, which are truncated and solved by iteration in numerical computations. The present solutions are satisfactory in comparison with other available results.

1. INTRODUCTION

Sandwich structures have gained great popularity in modern industries for their nature of lightweight and high strength. Neglect of transverse shear deformation causes the main disadvantage of the classical Kirchhoff plate theory in analysing behaviors of sandwich plates and shells. Reissner (1948, 1950) first proposed a theory governing finite deflections of sandwich plates with isotropic faces and cores. Alwan (1964) and Nowinski and Ohnabe (1973) extended Reissner's theory to large deflections of sandwich plates with orthotropic cores. On the basis of Reissner's theory, Alwan (1967) solved the nonlinear bending problem of rectangular sandwich plates by means of double trigonometric series with simply-supported stiffened edges. Under such a special boundary condition the solution is easy to obtain but of less use in practice. Kan and Huang (1967) applied the perturbation technique to the large deflection of rectangular sandwich plates with rigidly clamped edges. Kamiya (1976) extended Berger's assumption to nonlinear bending of sandwich plates and obtained solutions with hinged edges. Rao and Valsarajan (1983, 1986) analysed large deflection behavior of skew sandwich plates by parametric differentiation and integral-equation solution. Applying Galerkin's technique the finite deflection was investigated by Ng and Das (1986) for skew sandwich plates on an elastic foundation with rigidly clamped edges. Dutta and Banerjee (1991) studied nonlinear static and dynamic behaviors of sandwich plates based upon a new set of decoupled differential equations.

In view of existing results in nonlinear static and dynamic analysis of plates including transverse shearing deformation, analytic solutions by either energy methods or double series methods are based upon the form of assumed displacement, but for the limitation of which either the solutions are not satisfied by all boundary conditions or they are restricted too much with exception of at least one group of opposite edges simply supported. Reissner's theory of sandwich plates can be reduced to that of Kirchhoff's thin plates if the related parameter vanishes. However, it is contradictory that the solutions from Reissner's theory of sandwich plates reduce to those from Kirchhoff's theory of thin plates. In addition, because of the unknown degree of accuracy by using energy methods, analysis of high precision is required to accurately predict behaviors of structures in modern industries. It is necessary to seek analytic solutions of exact or quite accurate form.

Due to Reissner's omission of faces' bending effects there exist some problems unsolvable. Hoff (1950) set up a theory by considering the faces of sandwich plates as Kirchhoff thin plates instead of membranes in the Reissner's theory. The presentation extends Hoff's theory to nonlinear dynamic responses of sandwich plates in terms of Hamilton's principle, and deals with finite deflections of rectangular sandwich plates subjected to symmetrically distributed load.

As is well known, the Navier and Levy series are applied to solve the problems with all simply-supported edges or one group of opposite edges simply supported, respectively, but they are useless for other cases. The superposition principle efficiently used in linear analysis is invalid in nonlinear analysis. The presentation develops a new technique of mixed Fourier series for solving nonlinear problems. The mixed Fourier series are not a superposition of Navier and Levy series although the form is the sum of three series, whose coefficients are undetermined functions and undetermined constants of nonlinear internal relations. Solutions of large deflection equations of Karman type are formulated by making use of mixed Fourier series, whose coefficients are flexible to be determined by different boundary conditions. The resulting infinite set of simultaneous nonlinear algebraic equations is solved by iteration in numerical computations. The solutions are exact in the sense that they can be truncated to obtain any desired degree of accuracy. To the authors' knowledge, this is not found in the literature for nonlinear analysis of rectangular sandwich plates. Numerical results are graphically illustrated and discussed.

2. FUNDAMENTAL EQUATIONS AND BOUNDARY CONDITIONS

Under Hoff's assumptions that sandwich faces are considered as Kirchhoff thin plates and shear deformation is restricted to the core, displacements of a sandwich plate, whose midplane is on the xy plane and center at the origin of the Cartesian coordinate system, are expressed as follows:

$$u_1, u_3 = u \pm \frac{h+t_f}{2}\psi_x - \left(z \mp \frac{h+t_f}{2}\right)\frac{\partial w}{\partial x},$$

$$v_1, v_3 = v \pm \frac{h+t_f}{2}\psi_y - \left(z \mp \frac{h+t_f}{2}\right)\frac{\partial w}{\partial y}, \quad \left(\frac{h}{2} \leqslant |z| \leqslant \frac{h}{2}+t_f\right),$$

$$w_1, w_3 = w \tag{1}$$

for the upper and lower faces, and

$$u_2 = u + z\left(\frac{h+t_f}{h}\psi_x + \frac{t_f}{h}\frac{\partial w}{\partial x}\right),$$

$$v_2 = v + z\left(\frac{h+t_f}{h}\psi_y + \frac{t_f}{h}\frac{\partial w}{\partial y}\right), \quad \left(-\frac{h}{2} \leqslant z \leqslant \frac{h}{2}\right),$$

$$w_2 = w \tag{2}$$

for the core, where u, v and w are the inplane displacements and deflection of the midplane, ψ_x, ψ_y are the rotation angles of the midplane normal, h, t_f are the thickness of core and face of the sandwich plate.

By substitution of the foregoing expressions (1) and (2) into nonlinear strain–displacement relations of Karman type and then into Hooke's law for elastic isotropic materials, the strain energy of the sandwich plate, after integrating with respect to z, is obtained as

$$U_1, U_3 = \frac{t_f}{2E}\iint_S [(\sigma_x+\sigma_y)^2 + 2(1+\nu)(\tau_{xy}^2 - \sigma_x\sigma_y)]\,dx\,dy$$

$$\pm \frac{t_f(h+t_f)}{2}\iint_S \left[\sigma_x\frac{\partial \psi_x}{\partial x} + \sigma_y\frac{\partial \psi_y}{\partial y} + \tau_{xy}\left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x}\right)\right] dx\,dy$$

$$+\frac{D}{4}\iint_S\left[\left(\frac{\partial\psi_x}{\partial x}\right)^2+\left(\frac{\partial\psi_y}{\partial y}\right)^2+2v\frac{\partial\psi_x}{\partial x}\frac{\partial\psi_y}{\partial y}+\frac{1-v}{2}\left(\frac{\partial\psi_x}{\partial y}+\frac{\partial\psi_y}{\partial x}\right)^2\right]\mathrm{d}x\,\mathrm{d}y$$

$$+\frac{D_\mathrm{f}}{2}\iint_S\left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2+\left(\frac{\partial^2 w}{\partial y^2}\right)^2+2v\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}+2(1-v)\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2\right]\mathrm{d}x\,\mathrm{d}y, \tag{3a}$$

$$U_2=\frac{G_2(h+t_\mathrm{f})^2}{2h}\iint_S\left[\left(\psi_x+\frac{\partial w}{\partial x}\right)^2+\left(\psi_y+\frac{\partial w}{\partial y}\right)^2\right]\mathrm{d}x\,\mathrm{d}y, \tag{3b}$$

where D and D_f are, respectively, the effective bending stiffness of the sandwich plate and its face expressed as

$$D=\frac{Et_\mathrm{f}(h+t_\mathrm{f})^2}{2(1-v^2)},\quad D_\mathrm{f}=\frac{Et_\mathrm{f}^3}{12(1-v^2)}. \tag{4}$$

$\sigma_x, \sigma_y, \tau_{xy}$ are the effective inplane stresses of the sandwich plate expressed as

$$\sigma_x=\frac{E}{1-v^2}\left[\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{v}{2}\left(\frac{\partial w}{\partial y}\right)^2\right],$$

$$\sigma_y=\frac{E}{1-v^2}\left[\frac{\partial v}{\partial y}+v\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2+\frac{v}{2}\left(\frac{\partial w}{\partial x}\right)^2\right],$$

$$\tau_{xy}=G\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right). \tag{5}$$

E, v and G are the Young's modulus, Poisson ratio and shearing stiffness of faces, G_2 is the shearing stiffness of the core of the sandwich plate.

The external work, done by laterally arbitrarily distributed load q and uniform inplane biaxial compression p_x, p_y per unit length on the rectangular sandwich plate of $2a, 2b$ in length, is expressed in the form:

$$U_w=\iint_S qw\,\mathrm{d}x\,\mathrm{d}y-\left(\int_{-b}^{b}p_x u\,\mathrm{d}y\right)\Bigg|_{x=-a}^{a}-\left(\int_{-a}^{a}p_y v\,\mathrm{d}x\right)\Bigg|_{y=-b}^{b}. \tag{6}$$

The kinetic energy of the sandwich plate may be expressed as

$$U_{k1}, U_{k3}=\frac{\rho_\mathrm{f}}{2}\iint_S\left\{t_\mathrm{f}\left[\left(\frac{\partial u}{\partial t}\pm\frac{h+t_\mathrm{f}}{2}\frac{\partial\psi_x}{\partial t}\right)^2+\left(\frac{\partial v}{\partial t}\pm\frac{h+t_\mathrm{f}}{2}\frac{\partial\psi_y}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right]\right.$$

$$\left.+\frac{t_\mathrm{f}^3}{12}\left[\left(\frac{\partial^2 w}{\partial x\,\partial t}\right)^2+\left(\frac{\partial^2 w}{\partial y\,\partial t}\right)^2\right]\right\}\mathrm{d}x\,\mathrm{d}y, \tag{7a}$$

$$U_{k2}=\frac{\rho_\mathrm{c}}{2}\iint_S\left\{h\left[\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial v}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right]+\frac{h^3}{12}\left[\left(\frac{h+t_\mathrm{f}}{h}\frac{\partial\psi_x}{\partial t}+\frac{t_\mathrm{f}}{h}\frac{\partial^2 w}{\partial x\,\partial t}\right)^2\right.\right.$$

$$\left.\left.+\left(\frac{h+t_\mathrm{f}}{2}\frac{\partial\psi_y}{\partial t}+\frac{t_\mathrm{f}}{h}\frac{\partial^2 w}{\partial y\,\partial t}\right)^2\right]\right\}\mathrm{d}x\,\mathrm{d}y, \tag{7b}$$

where ρ_f and ρ_c are the mass densities of the faces and core of the sandwich plate, t is the time parameter.

Usually the effect of inplane motion is negligibly small compared with transverse motion in investigating transverse deformation of plates, and usually in the sense of Karman large deflection type the omission of rotational motion of faces is also reasonable for their smaller thickness. As a result, the total kinetic energy of the sandwich plate can be simplified as

$$U_k = U_{k1}+U_{k2}+U_{k3} = \iint_S \left\{(\rho_f t_f+\tfrac{1}{2}\rho_c h)\left(\frac{\partial w}{\partial t}\right)^2 + \tfrac{1}{4}(\rho_f t_f+\tfrac{1}{6}\rho_c h)(h+t_f)^2\left[\left(\frac{\partial \psi_x}{\partial t}\right)^2+\left(\frac{\partial \psi_y}{\partial t}\right)^2\right]\right\} \mathrm{d}x\,\mathrm{d}y. \quad (8)$$

The dynamic fundamental equations and boundary conditions are now derived from Hamilton's principle by use of expressions (3), (6) and (8) and, along with the compatibility condition for the inplane displacements of midplane, given in the following dimensionless form:

$$\frac{\partial^2\Psi_\xi}{\partial\xi^2}+\frac{1-v}{2}\lambda^2\frac{\partial^2\Psi_\xi}{\partial\eta^2}+\frac{1+v}{2}\lambda\frac{\partial^2\Psi_\eta}{\partial\xi\,\partial\eta}-\frac{1}{\varepsilon}\left(\Psi_\xi+\frac{\partial W}{\partial\xi}\right)-J\frac{\partial^2\Psi_\xi}{\partial\tau^2}=0, \quad (9)$$

$$\frac{1+v}{2}\lambda\frac{\partial^2\Psi_\xi}{\partial\xi\,\partial\eta}+\frac{1-v}{2}\frac{\partial^2\Psi_\eta}{\partial\xi^2}+\lambda^2\frac{\partial^2\Psi_\eta}{\partial\eta^2}-\frac{1}{\varepsilon}\left(\Psi_\eta+\lambda\frac{\partial W}{\partial\eta}\right)-J\frac{\partial^2\Psi_\eta}{\partial\tau^2}=0, \quad (10)$$

$$\frac{\partial\Psi_\xi}{\partial\xi}+\lambda\frac{\partial\Psi_\eta}{\partial\eta}+\mathscr{L}_1 W-\varepsilon_f\mathscr{L}_1^2 W-\varepsilon R\left(\frac{\partial^2 W}{\partial\xi^2}+\rho\lambda^2\frac{\partial^2 W}{\partial\eta^2}\right)-\varepsilon\frac{\partial^2 W}{\partial\tau^2} = -\varepsilon(Q+F), \quad (11)$$

$$\mathscr{L}_1^2\Phi = -\frac{1-v^2}{2}\lambda^2 A, \quad (12)$$

$$\frac{\partial^2 U}{\partial\xi^2}+\frac{1-v}{2}\lambda^2\frac{\partial^2 U}{\partial\eta^2}+\frac{1+v}{2}\lambda\frac{\partial^2 V}{\partial\xi\,\partial\eta} = -B, \quad (13)$$

$$\frac{1+v}{2}\lambda\frac{\partial^2 U}{\partial\xi\,\partial\eta}+\frac{1-v}{2}\frac{\partial^2 V}{\partial\xi^2}+\lambda^2\frac{\partial^2 V}{\partial\eta^2} = -C, \quad (14)$$

$$W=0,\quad \varepsilon_f\frac{\partial W}{\partial\xi}=0,\quad \Psi_\xi=0,\quad \Psi_\eta=0,\quad \frac{\partial^2\Phi}{\partial\eta^2}=0,\quad \frac{\partial^2\Phi}{\partial\xi\,\partial\eta}=0 \quad (15)$$

at $\xi = -1$ or 1 for a loosely clamped edge

$$W=0,\quad \varepsilon_f\frac{\partial^2 W}{\partial\xi^2}=0,\quad \frac{\partial\Psi_\xi}{\partial\xi}=0,\quad \Psi_\eta=0,\quad \frac{\partial^2\Phi}{\partial\eta^2}=0,\quad \frac{\partial^2\Phi}{\partial\xi\,\partial\eta}=0 \quad (16)$$

at $\xi = -1$ or 1 for a simply-supported edge

$$W=0,\quad \varepsilon_f\frac{\partial W}{\partial\xi}=0,\quad \Psi_\xi=0,\quad \Psi_\eta=0,\quad U=0,\quad V=0 \quad (17)$$

at $\xi = -1$ or 1 for a rigidly clamped edge

$$W=0,\quad \varepsilon_f\frac{\partial^2 W}{\partial\xi^2}=0,\quad \frac{\partial\Psi_\xi}{\partial\xi}=0,\quad \Psi_\eta=0,\quad U=0,\quad V=0 \quad (18)$$

at $\xi = -1$ or 1 for a hinged edge

$$W = 0, \quad \varepsilon_f \frac{\partial W}{\partial \eta} = 0, \quad \Psi_\xi = 0, \quad \Psi_\eta = 0, \quad \frac{\partial^2 \Phi}{\partial \xi^2} = 0, \quad \frac{\partial^2 \Phi}{\partial \xi \, \partial \eta} = 0 \tag{19}$$

at $\eta = -1$ or 1 for a loosely clamped edge

$$W = 0, \quad \varepsilon_f \frac{\partial^2 W}{\partial \eta^2} = 0, \quad \Psi_\xi = 0, \quad \frac{\partial \Psi_\eta}{\partial \eta} = 0, \quad \frac{\partial^2 \Phi}{\partial \xi^2} = 0, \quad \frac{\partial^2 \Phi}{\partial \xi \, \partial \eta} = 0 \tag{20}$$

at $\eta = -1$ or 1 for a simply-supported edge

$$W = 0, \quad \varepsilon_f \frac{\partial W}{\partial \eta} = 0, \quad \Psi_\xi = 0, \quad \Psi_\eta = 0, \quad U = 0, \quad V = 0 \tag{21}$$

at $\eta = -1$ or 1 for a rigidly clamped edge

$$W = 0, \quad \varepsilon_f \frac{\partial^2 W}{\partial \eta^2} = 0, \quad \Psi_\xi = 0, \quad \frac{\partial \Psi_\eta}{\partial \eta} = 0, \quad U = 0, \quad V = 0 \tag{22}$$

at $\eta = -1$ or 1 for a hinged edge with only some usual boundary conditions listed as above, where the differential operator $\mathscr{L}_1$ is

$$\mathscr{L}_1 = \frac{\partial^2}{\partial \xi^2} + \lambda^2 \frac{\partial^2}{\partial \eta^2} \tag{23}$$

and the nonlinear terms F, A, B and C are given by

$$F = \lambda^2 \mathscr{L}_2(W, \Phi), \tag{24}$$

$$A = \mathscr{L}_2(W, W), \tag{25}$$

$$\mathscr{L}_2(W, \Phi) = \frac{\partial^2 W}{\partial \xi^2} \frac{\partial^2 \Phi}{\partial \eta^2} - 2 \frac{\partial^2 W}{\partial \xi \, \partial \eta} \frac{\partial^2 \Phi}{\partial \xi \, \partial \eta} + \frac{\partial^2 W}{\partial \eta^2} \frac{\partial^2 \Phi}{\partial \xi^2}, \tag{26}$$

$$F = \left[\frac{\partial U}{\partial \xi} + \frac{1}{2}\left(\frac{\partial W}{\partial \xi}\right)^2\right]\left(\frac{\partial^2 W}{\partial \xi^2} + \nu \lambda^2 \frac{\partial^2 W}{\partial \eta^2}\right) + \lambda\left[\frac{\partial V}{\partial \eta} + \frac{\lambda}{2}\left(\frac{\partial W}{\partial \eta}\right)^2\right]\left(\lambda^2 \frac{\partial^2 W}{\partial \eta^2} + \nu \frac{\partial^2 W}{\partial \xi^2}\right)$$
$$+ (1-\nu)\lambda\left(\lambda \frac{\partial U}{\partial \eta} + \frac{\partial V}{\partial \xi} + \lambda \frac{\partial W}{\partial \xi} \frac{\partial W}{\partial \eta}\right) \frac{\partial^2 W}{\partial \xi \, \partial \eta}, \tag{27}$$

$$B = \frac{\partial W}{\partial \xi}\left(\frac{\partial^2 W}{\partial \xi^2} + \frac{1-\nu}{2} \lambda^2 \frac{\partial^2 W}{\partial \eta^2}\right) + \frac{1+\nu}{2} \lambda^2 \frac{\partial W}{\partial \eta} \frac{\partial^2 W}{\partial \xi \, \partial \eta}, \tag{28}$$

$$C = \lambda \frac{\partial W}{\partial \eta}\left(\lambda^2 \frac{\partial^2 W}{\partial \eta^2} + \frac{1-\nu}{2} \frac{\partial^2 W}{\partial \xi^2}\right) + \frac{1+\nu}{2} \lambda \frac{\partial W}{\partial \xi} \frac{\partial^2 W}{\partial \xi \, \partial \eta}. \tag{29}$$

Usually the expressions (24)–(26) and (27)–(29) are used in the problems with movable and immovable boundary conditions, respectively.

The above dimensionless parameters are defined as

$$\lambda = \frac{a}{b}, \quad \xi = \frac{x}{a}, \quad \eta = \frac{y}{b}, \quad \Psi_\xi = \frac{2a}{h+t_f}\psi_x, \quad \Psi_\eta = \frac{2a}{h+t_f}\psi_y, \quad W = \frac{2}{h+t_f}w,$$

$$Q = \frac{2a^4}{D(h+t_f)}q, \quad R = \frac{a^2}{D}p_x, \quad \rho = \frac{p_y}{p_x}, \quad \Phi = \frac{R}{2\lambda^2}(\eta^2+\rho\lambda^2\xi^2)+\frac{2t_f}{D}\varphi,$$

$$U = \frac{4a}{(h+t_f)^2}u, \quad V = \frac{4a}{(h+t_f)^2}v, \quad \varepsilon = \frac{Dh}{G_2(h+t_f)^2a^2}, \quad \varepsilon_f = \frac{2D_fh}{G_2(h+t_f)^2a^2},$$

$$J = \frac{(6\rho_f t_f+\rho_c h)(h+t_f)^2}{12(2\rho_f t_f+\rho_c h)a^2}, \quad \tau = \sqrt{\frac{D}{(2\rho_f t_f+\rho_c h)a^4}}\,t, \tag{30}$$

where φ is the stress function defined by

$$\sigma_x = \frac{\partial^2\varphi}{\partial y^2}, \quad \sigma_y = \frac{\partial^2\varphi}{\partial x^2}, \quad \tau_{xy} = -\frac{\partial^2\varphi}{\partial x\,\partial y}. \tag{31}$$

For a moderately thick plate of Reissner–Mindlin type, the dimensionless governing equations and boundary conditions are the same as those for the Reissner's sandwich plate. They can be obtained from those for the Hoff's sandwich plate by setting $\varepsilon_f = 0$, and hence the second of each set of boundary conditions (15)–(22) disappearing automatically, and be further reduced to those for the Kirchhoff thin plate by setting $\varepsilon = 0$. The dimensionless parameters for the moderately thick plate can be obtained directly from (30) by taking $D_f = 0$, $\rho_c = 0$ and replacing $\rho_f, h+t_f, t_f$, and G_2 with $\rho_0, h/\sqrt{3}$, $h/2$, and $(\sqrt{3}-3/2)\kappa G$, where ρ_0, h are the mass density and thickness of the moderately thick plate, G is the shearing stiffness, κ is the shearing correction factor, being 5/6 in the Reissner's theory, or $\pi^2/12$ in Mindlin's theory.

3. SOLUTIONS OF NONLINEAR BENDING OF SANDWICH PLATES

As an example, nonlinear bending of rectangular sandwich plates of Hoff type under transverse symmetric load is considered herein. By taking $R = 0$, $\rho = 0$ and dropping the terms concerned with time, eqns (9)–(11) may be rewritten as follows:

$$\mathscr{L}[\Psi_\xi \quad \Psi_\eta \quad W]^T = [0 \quad 0 \quad -\varepsilon(Q+F)]^T, \tag{32}$$

where $\mathscr{L}$ is the partial differential operator defined by

$$\mathscr{L} = \begin{bmatrix} \frac{\partial^2}{\partial\xi^2}+\frac{1-\nu}{2}\lambda^2\frac{\partial^2}{\partial\eta^2}-\frac{1}{\varepsilon} & \frac{1+\nu}{2}\lambda\frac{\partial^2}{\partial\xi\,\partial\eta} & -\frac{1}{\varepsilon}\frac{\partial}{\partial\xi} \\ \frac{1+\nu}{2}\lambda\frac{\partial^2}{\partial\xi\,\partial\eta} & \frac{1-\nu}{2}\frac{\partial^2}{\partial\xi^2}+\lambda^2\frac{\partial^2}{\partial\eta^2}-\frac{1}{\varepsilon} & -\frac{1}{\varepsilon}\lambda\frac{\partial}{\partial\eta} \\ \frac{\partial}{\partial\xi} & \lambda\frac{\partial}{\partial\eta} & \mathscr{L}_1-\varepsilon_f\mathscr{L}_1^2 \end{bmatrix}. \tag{33}$$

Assume that the dimensionless rotation angles Ψ_ξ, Ψ_η of midplane normal and deflection W are of the following mixed Fourier series form:

$$\begin{pmatrix}\Psi_\xi\\ \Psi_\eta\\ W\end{pmatrix} = \sum_{n=1}^{\infty}\begin{pmatrix}G_n(\xi)\cos\beta_n\eta\\ S_n(\xi)\sin\beta_n\eta\\ X_n(\xi)\cos\beta_n\eta\end{pmatrix} + \sum_{m=1}^{\infty}\begin{pmatrix}H_m(\eta)\sin\beta_m\xi\\ T_m(\eta)\cos\beta_m\xi\\ Y_m(\eta)\cos\beta_m\xi\end{pmatrix} + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{pmatrix}\Psi^*_{\xi mn}\sin\beta_m\xi\cos\beta_n\eta\\ \Psi^*_{\eta mn}\cos\beta_m\xi\sin\beta_n\eta\\ W^*_{mn}\cos\beta_m\xi\cos\beta_n\eta\end{pmatrix} \tag{34}$$

in which

$$\beta_i = (i-1)\pi. \tag{35}$$

The mixed Fourier series are different from the conventional Navier or Levy series and not simply superpositions. Their coefficients are unknown functions and unknown constants of nonlinear internal relations and may be satisfied by different boundary conditions.

Substituting expressions (34) into eqns (32) and making the following equations valid:

$$\mathscr{L}\begin{bmatrix}\sum_{n=1}^{\infty} G_n(\xi)\cos\beta_n\eta & \sum_{m=1}^{\infty} H_m(\eta)\sin\beta_m\xi & \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\Psi^*_{\xi mn}\sin\beta_m\xi\cos\beta_n\eta\\ \sum_{n=1}^{\infty} S_n(\xi)\sin\beta_n\eta & \sum_{m=1}^{\infty} T_m(\eta)\cos\beta_m\xi & \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\Psi^*_{\eta}\cos\beta_m\xi\sin\beta_n\eta\\ \sum_{n=1}^{\infty} X_n(\xi)\cos\beta_n\eta & \sum_{m=1}^{\infty} Y_m(\eta)\cos\beta_m\xi & \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W^*_{mn}\cos\beta_m\xi\cos\beta_n\eta\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\varepsilon(Q+F)\end{bmatrix}. \tag{36}$$

According to orthogonality properties of trigonometric series, the above equations can be reduced to ordinary differential equations and linear algebraic equations, whose solutions can be obtained without difficulty. With this, using the first four of the boundary conditions (15)–(22) results in expressions of Ψ_ξ, Ψ_η and W. Similar to the above procedure, the solutions of Φ, U and V can also be obtained. These algebraic details may be easily performed by using computerized symbolic manipulator systems such as MACSYMA or MATHEMATICA and thus are not given herein.

Therefore, the fundamental equations and boundary conditions are reduced to an infinite set of simultaneous nonlinear algebraic equations. These equations are to be solved for unknown coefficients for any given set of values of plate parameters ν, λ, ε, ε_f, and load parameter Q. As soon as these coefficients are determined, the deflection, bending stresses and membrane stresses of plates can be found. By combined choice of boundary conditions, the nonlinear bending problem of rectangular Hoff type sandwich plates can be solved for a number of different movable and immovable boundary conditions under lateral symmetric load.

4. NUMERICAL COMPUTATIONS AND DISCUSSION OF RESULTS

Numerical computations were performed on the VAX8700 digital computer. Each summation was truncated to the first six terms. Such precision is sufficient, taking into account drawing errors. The system of nonlinear algebraic equations is solved by an iterative procedure. The value of the deflection coefficient W_{11} is prescribed and an initial guess of other deflection coefficients W_{mn} is made. With W_{mn}, other unknown coefficients and new W_{mn} (except W_{11}), Q are then calculated for the first iteration. These values of W_{mn} and the prescribed value of W_{11} are now used as new iterative initial values. The process is repeated until the desired accuracy is achieved, and thus the nonlinear bending problem of rectangular plates is solved for movable and immovable boundary conditions. The criterion for the convergence of the iterative process is that the difference between the final value of the external load and the average of the values in the previous three iterations is less than 0.1%. As long as good initial values are given, the iteration converges rapidly, generally within 10 iterations.

Numerical results are graphically presented. On each curve shown in Figs 6 and 7 boundary conditions are denoted by the symbols C_L (loosely clamped), S (simply supported), C_R (rigidly clamped), H (hinged) at the edges $\xi = -1, \eta = -1, \xi = 1, \eta = 1$ in the sequence.

The central deflection of a rectangular sandwich plate is plotted against external load in Figs 1–5, of which the first four correspond to uniform pressure and the last one to the

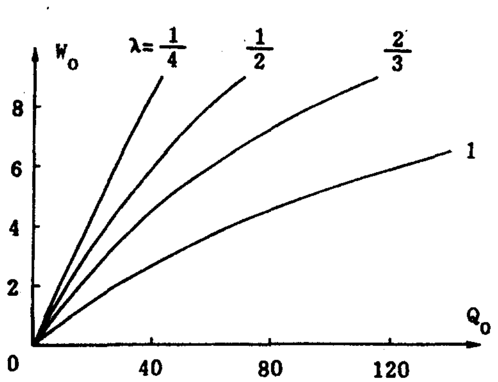


Fig. 1. Load–deflection curves for simply-supported rectangular sandwich plates under uniform pressure with different aspect ratios ($\nu = 0.32$, $E = 10^7$ psi, $G_2 = 10^4$ psi, $t_f/a = 0.00125$, $h/a = 0.05$).

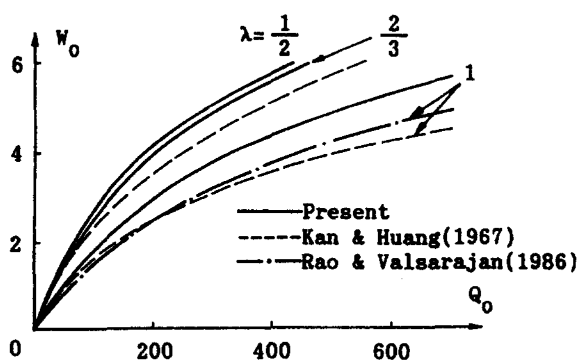


Fig. 2. Load–deflection curves for rigidly clamped rectangular sandwich plates under uniform pressure with different aspect ratios ($\nu = 0.3$, $E = 1.05 \times 10^7$ psi, $G_2 = 5 \times 10^4$ psi, $t_f/a = 0.0006$, $h/a = 0.04$).

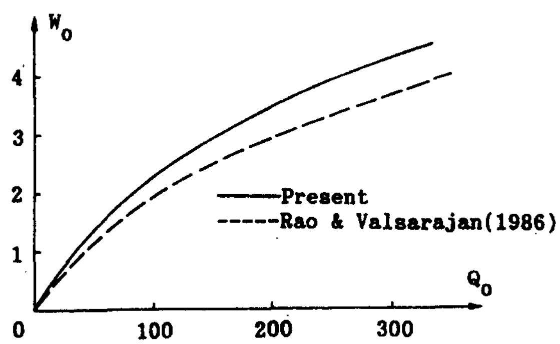


Fig. 3. Load–deflection curves for rigidly clamped square sandwich plates under uniform pressure ($\nu = 0.32$, $E = 10^7$ psi, $G_2 = 10^4$ psi, $t_f/a = 0.00125$, $h/a = 0.05$).

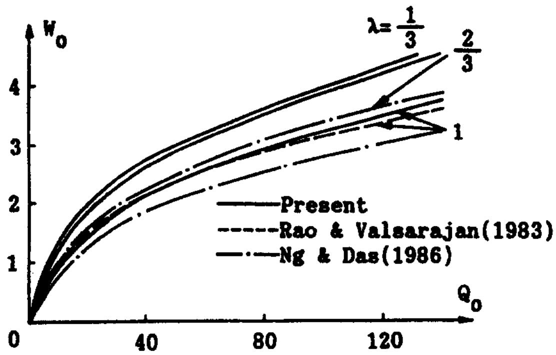


Fig. 4. Load–deflection curves for rigidly clamped rectangular sandwich plates under uniform pressure with different aspect ratios ($\nu = 0.32$, $E = 10^7$ psi, $G_2 = 10^3$ psi, $t_f/a = 0.00125$, $h/a = 0.05$).

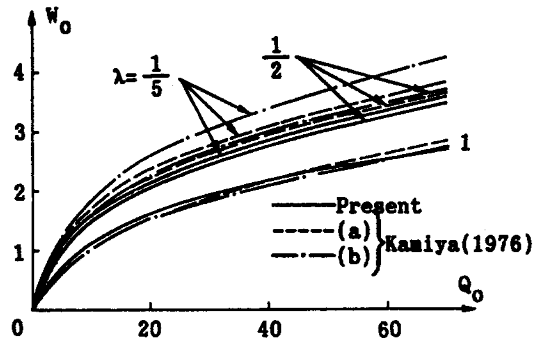


Fig. 5. Load–deflection curves for hinged rectangular sandwich plates under cosine distributed pressure with different aspect ratios ($\nu = 0.3$, $E = 1.045 \times 10^7$ psi, $G_2 = 6 \times 10^3$ psi, $t_f/a = 0.005$, $h/a = 0.12992$).

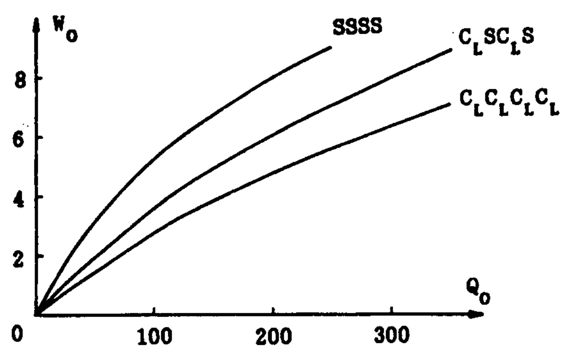


Fig. 6. Load–deflection curves for square sandwich plates under uniform pressure with different movable edges ($\nu = 0.32$, $E = 10^7$ psi, $G_2 = 10^4$ psi, $t_f/a = 0.00125$, $h/a = 0.05$).

pressure of cosine form $q = q_0 \cos[\pi x/(2a)] \cos[\pi y/(2b)]$. It is observed from Figs 1, 2, 4, 5 that for a given dimensionless pressure the central deflection increases with decreasing aspect ratio. The curves in Fig. 1 straighten with decreasing aspect ratio. In fact, when the aspect ratio is zero the nonlinear effect vanishes for a simply-supported infinite long plate and the relation between the load and the central deflection becomes linear. The $\lambda < 1/2$ curve for a rigidly clamped rectangular plate approaches the $\lambda = 1/2$ curve and is not presented in Fig. 2. Therefore, such a rectangular plate of $\lambda \leqslant 1/2$ can be treated as an infinitely long plate. The same conclusion is also adaptive for a rigidly clamped plate of $\lambda \leqslant 1/3$ in Fig. 4.

A common drawback in the research findings (Kan and Huang, 1967; Rao and Valsarajan, 1986) is that the sandwich plates are restricted too much by unnecessary boundary conditions in applying Reissner's theory. It is seen that larger differences exist between the present results and the results given by Kan and Huang (1967) and Rao and Valsarajan (1986) for $\lambda = 1$ in Figs 2 and 3, and smaller differences exist for $\lambda = 2/3$. The $\lambda = 1/2$ curve in the reference (Kan and Huang, 1967) nearly coincides with the present result with the same aspect ratio and is not presented in Fig. 2. This arises from the fact that the effect on central deflections of unnecessary boundary restriction weakens with decreasing aspect ratio. In addition to the above, the inaccuracy of the results of Kan and Huang (1967) may also result from the adopted procedure of solution and be further removed by increasing the perturbation order and adding more terms to the displacement polynomials (Chia, 1972). The results of Rao and Valsarajan (1983) and Ng and Das (1986) have the same drawback as the results of Kan and Huang (1967) and Rao and Valsarajan (1986). A comparison of the present results with the results given by Rao and Valsarajan (1983) and Ng and Das (1986) is plotted in Fig. 4, where the $\lambda = 1/3$ curve in Ng and Das (1986) nearly coincides with the $\lambda = 2/3$ curve in the same reference but is not given. Nonlinear behavior of hinged rectangular sandwich plates under cosine distributed pressure was investigated by Kamiya (1976) based upon the large deflection theory of (a) Karman type; and (b) extension of Berger's approximation to sandwich plates. Good agreement is seen in Fig. 5 between the present results and the two resuls for $\lambda = 1$, and their disagreement to a certain degree increases with decreasing aspect ratio, especially for Berger's approximation.

Figures 6 and 7 show the relations between the central deflection of square sandwich plates and lateral uniform pressure with movable and immovable symmetric boundary conditions. For a fixed dimensionless load the central deflection increases with the lessening of the rotation restraint on the edges.

To indicate the effect of including and neglecting bending stiffness of facing plates, Figs 8 (a–c) are presented for a variety of configurations of rigidly clamped sandwich plates, with the $\varepsilon_f/\varepsilon = 0$ curves corresponding to the results from Reissner's theory. Calculations show an evident distinction between the present result and that of Reissner's theory. For a given dimensionless load the central deflection predicted by Reissner's theory is higher than that predicted by the present theory, apparently this results from bending capacity of facing plates. For different aspect ratios $\lambda = 1$, $1/2$ and $1/4$, the relative errors of results from different theories are, on the whole, about 5% for $\varepsilon = 0.1$, from 7 to 12% for $\varepsilon = 0.4$, and from 8 to 16% for $\varepsilon = 0.7$. It implies that the extended Hoff's theory is needed in nonlinear analysis of rigidly clamped sandwich plates. The relative errors for other boundary conditions decrease with the loosening of the restraint on edges and results are not plotted herein. The relative errors for simply-supported sandwich plates are in general within 3% and hence Reissner's theory is adaptable in such a case, especially for those sandwich plates with smaller ε.

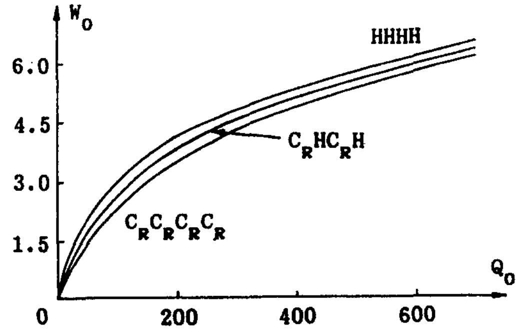
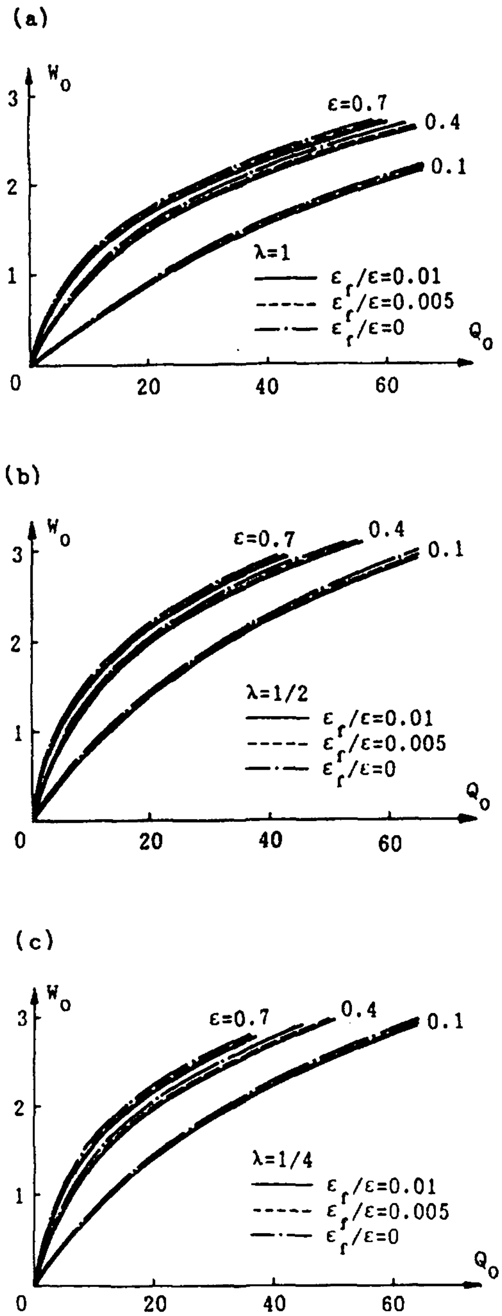


Fig. 7. Load–deflection curves for square sandwich plates under uniform pressure with different immovable edges ($\nu = 0.32$, $E = 10^7$ psi, $G_2 = 10^4$ psi, $t_f/a = 0.00125$, $h/a = 0.05$).



Figs 8 (a–c). Load-deflection curves for a variety of configurations of rigidly clamped sandwich plates under uniform pressure.

5. CONCLUSIONS

The Hoff type theory underlying nonlinear dynamic responses of sandwich plates is derived from Hamilton's principle. Nonlinear fundamental equations and boundary conditions of dimensionless form are unified with those for moderately thick plates of Reissner–Mindlin type and Reissner's sandwich plates. Finite deflections of rectangular sandwich plates of the Hoff type are studied with some symmetric boundary conditions. By developing a new technique of mixed Fourier series in nonlinear analysis exact solutions of mixed series form are obtained and they can be truncated to approach any desired degree of accuracy. The present solutions are satisfactory in comparison with other available results from the Reissner's theory. Bending stresses of faces of sandwich plates, unsolvable from the

Reissner's theory, may be numerically calculated without difficulty from present analytic formulae if required. The present results can be reduced to those for moderately thick plates of Reissner–Mindlin type, Reissner's sandwich plates, and Kirchhoff thin plates without contradictions. Analysis of nonlinear dynamic response of rectangular moderately thick plates and sandwich plates will be presented in the authors' subsequent paper.

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